or r_c. A more detailed discussion about the skewness of I_A is included in [12], mainly in Section III.

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Novel Partial Selection Schemes for AF Relaying in Nakagami-m Fading Channels

Yunfei Chen, *Senior Member, IEEE*, Cheng-Xiang Wang, *Senior Member, IEEE*, Hailin Xiao, and Dongfeng Yuan, *Senior Member, IEEE*

***Abstract*—New partial relay selection schemes for cooperative diversity based on amplify-and-forward (AF) relaying are proposed in Nakagami-m fading channels. Their performances are compared with the conventional partial selection scheme. Numerical results show that the new schemes have performance gains of up to 5 dB over the conventional scheme. In some cases, their performances are indistinguishable from the full selection scheme, but they have much simpler structures. Numerical results also show that it is more important to choose the idle user for the hop with a small average signal-to-noise ratio (SNR) or an m parameter in partial selection. Based on this observation, a new adaptive partial selection scheme based on the average SNR, and the m parameter is derived. A complexity analysis also shows that the new schemes reduce the complexity in some cases.**

***Index Terms*—Amplify-and-forward (AF), performance analysis, user selection.**

I. Introduction

In recent years, cooperative diversity has been proposed as an effective method of improving the performance of a wireless system [1]. In a cooperative diversity system, idle users are employed to forward signals from the source to the destination. The idle users act as virtual antennas to achieve cooperative space diversity at the destination, in contrast to the traditional diversity system where multiple antennas are physically installed at the destination [2]–[5]. Among all the existing protocols for cooperative diversity, amplify-and-forward (AF) relaying is one of the simplest protocols [1]. The performance of AF cooperative diversity improves as the number of idle users increases [6]. However, the complexity of the network also increases as the number of the idle users increases. In some applications,

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Y. Chen is with the School of Engineering, University of Warwick, Coventry CV4 7AL, U.K. (e-mail: Yunfei.Chen@warwick.ac.uk).

C.-X. Wang is with the Joint Research Institute for Signal and Image Processing, School of Engineering and Physical Sciences, Heriot-Watt University, Edinburgh EH14 4AS, U.K. (e-mail: Cheng-Xiang.Wang@hw.ac.uk).

H. Xiao is with the School of Information and Communication, Guilin University of Electronic Technology, Guilin 541004, China (e-mail: xhl_xiaohailin@yahoo.com.cn).

D. Yuan is with the School of Information Science and Engineering, Shandong University, Jinan 250100, China (e-mail: dfyuan@sdu.edu.cn).

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such as wireless sensor networks, complexity is more important than performance in the performance–complexity design tradeoff for wireless systems to achieve long battery life once the minimum performance requirement is met. To reduce the network complexity in these applications, user selection is implemented that often chooses one out of all available idle users for AF cooperative diversity.

In [7], the optimal full selection scheme was proposed by choosing the idle user with the largest instantaneous end-to-end signal-to-noise ratio (SNR). In [8] and [9], two suboptimal full selection schemes were proposed by choosing the idle user with the largest harmonic mean or the minimum of the instantaneous SNRs of the first and second hops. In [10], a subset of idle users was chosen by comparing their combined instantaneous SNR with a preset threshold. In [11], time-varying channels were considered for relay selection. All these selection schemes require knowledge of the instantaneous SNRs of both the first and second hops for each idle user. To reduce complexity, [12] proposed a partial selection scheme that only compares the instantaneous SNR of the first hop for each idle user in Rayleigh fading channels. In [13] and [14], the scheme proposed in [12] was evaluated by considering the feedback delay and multiple antennas at the destination, respectively.

In this paper, we propose three new partial selection schemes for AF cooperative diversity with variable relay gain. The exact expression for the error rate of the first new scheme is analytically derived, whereas the error rates of the second and third new schemes are calculated via simulations. Moreover, we derive the exact expressions for the error rates of the optimal full selection scheme and the conventional partial selection scheme for Nakagami-m fading channels. To the best of the authors' knowledge, these are not available in the literature. Numerical results show that the new partial selection schemes have performance gains of up to 5 dB over the conventional partial selection scheme, and in some cases, their performances are very close to that of the optimal full selection scheme, but they have much simpler structures. It is also shown that choosing the best idle user for the hop with a smaller average SNR or an m parameter is important. Based on this, we also propose a new adaptive partial selection scheme by using the average SNR and the Nakagami m parameter, which can be estimated using results in [15].

II. Relay Selection

Similar to [12], consider an AF cooperative diversity system with one source, one destination, and N relays. There is no direct link between the source and the destination. The idle user links have two-hop transmissions. In the first time slot, the source transmits the signal to the idle users such that the received signal at the kth idle user can be expressed as

$$u_k(t) = h_{1,k}\sqrt{E_1}x(t) + n_{1,k}(t) \tag{1}$$

where $k = 1, 2, \ldots, N$ is the user index, $h_{1,k}$ is the complex fading gain in the channel between the source and the kth idle user, E_1 is the transmitted signal energy, $x(t)$ is the transmitted signal, and $n_{1,k}(t)$ is the complex Gaussian noise in the channel between the source and the kth idle user with noise power $N_{1,k}$. In the second time slot, the received signals at the idle users are amplified and transmitted such that the received signal from the kth idle user at the destination is

$$y_k(t) = h_{2,k}\alpha_k u_k(t) + n_{2,k}(t) \tag{2}$$

where $h_{2,k}$ is the complex fading gain in the channel between the kth idle user and the destination, $\alpha_k = \sqrt{E_{2,k}/(E_1|h_{1,k}|^2 + N_{1,k})}$ is the amplification factor, $E_{2,k}$ is the radiated energy at the kth idle user, and $n_{2,k}(t)$ is the complex Gaussian noise in the channel between the kth idle user and the destination with noise power $N_{2,k}$. All the links experience Nakagami-m fading such that $|h_{1,k}|$ follows a Nakagami distribution with $E\{|h_{1,k}|^2\} = \Omega_{1,k}$ and m parameter $m_{1,k}$, whereas $|h_{2,k}|$ follows a Nakagami distribution with $E\{|h_{2,k}|^2\} = \Omega_{2,k}$ and m parameter $m_{2,k}$. In this paper, it is assumed that $E_{2,k} = E_2$, $N_{1,k} = N_1$, $N_{2,k} = N_2$, $\Omega_{1,k} = \Omega_1$, $\Omega_{2,k} = \Omega_2$, $m_{1,k} = m_1$, and $m_{2,k} = m_2$ for $k = 1, 2, \ldots, N$, similar to [12]. The instantaneous end-to-end SNR of the kth link can be shown as $\gamma_k = \gamma_{1,k}\gamma_{2,k}/(\gamma_{1,k} + \gamma_{2,k} + 1)$, where $\gamma_{1,k} = |h_{1,k}|^2 E_1/N_1$ and $\gamma_{2,k} = |h_{2,k}|^2 E_2/N_2$ are the instantaneous SNRs of the first and second hops, respectively. In Nakagami-m fading channels, $\gamma_{1,k}$ follows a Gamma distribution with shape parameter m_1 and scale parameter $\bar{\gamma}_1/m_1$, whereas $\gamma_{2,k}$ follows a Gamma distribution with shape parameter m_2 and scale parameter $\bar{\gamma}_2/m_2$, where $\bar{\gamma}_1 = \Omega_1 E_1/N_1$ and $\bar{\gamma}_2 = \Omega_2 E_2/N_2$ are the average SNRs of the first and second hops, respectively.

A. Optimal Full Selection Scheme

In the optimal full selection scheme, the idle user is selected according to

$$K = \max_{k=1,2,\ldots,N}\{\gamma_k\}. \tag{3}$$

Denote this scheme as the $\max\{\gamma_k\}$ scheme. Using (3), the error rate can be derived as

$$P_e = \int_0^\infty P(e|x) f_{\gamma_K}(x)dx = \int_0^\infty P(e|x)dF_{\gamma_K}(x) \tag{4}$$

where $P(e|x)$ is the conditional probability of error, which is conditioned on γ_K, and $f_{\gamma_K}(x)$ and $F_{\gamma_K}(x)$ are the probability density function (pdf) and the cumulative distribution function (cdf) of $\gamma_K = \max\{\gamma_1, \gamma_2, \ldots, \gamma_N\}$, respectively. Since $\gamma_1, \gamma_2, \ldots, \gamma_N$ are independent and identically distributed, one has $F_{\gamma_K}(x) = F^N_{\gamma_k}(x)$, where $F_{\gamma_k}(x)$ is the cdf of γ_k given by [16, eq. (2)]

$$\begin{aligned} F_{\gamma_k}(x) = 1 - &\frac{2m_2^{m_2}(m_1-1)!e^{-\frac{m_1}{\bar{\gamma}_1}x - \frac{m_2}{\bar{\gamma}_2}x}}{\bar{\gamma}_2^{m_2}\Gamma(m_1)\Gamma(m_2)} \\ &\times \sum_{i_1=0}^{m_1-1}\sum_{i_2=0}^{i_1}\sum_{i_3=0}^{m_2-1} \frac{\binom{i_1}{i_2}\binom{m_2-1}{i_3}}{i_1!}\left(\frac{m_2}{\bar{\gamma}_2}\right)^{\frac{i_2-i_3-1}{2}} \\ &\cdot \left(\frac{m_1}{\bar{\gamma}_1}\right)^{\frac{2i_1-i_2+i_3+1}{2}} x^{\frac{2i_1+2m_2-i_2-i_3-1}{2}} \\ &\cdot (x+1)^{\frac{i_2+i_3+1}{2}} K_{i_2-i_3-1}\left(2\sqrt{\frac{m_1 m_2 x(1+x)}{\bar{\gamma}_1\bar{\gamma}_2}}\right) \end{aligned} \tag{5}$$

with $K_{i_2-i_3-1}(\cdot)$ being the $(i_2 - i_3 - 1)$th-order modified Bessel function of the second kind [17, 8.432]. From (4), one has

$$P_e = \int_0^\infty P(e|x)dF^N_{\gamma_k}(x). \tag{6}$$

Using integration by parts, one further has

$$P_e = P(e|x)F^N_{\gamma_k}(x)_0^\infty - \int_0^\infty F^N_{\gamma_k}(x)dP(e|x). \tag{7}$$

For binary phase-shift keying (BPSK), one has $P(e|x) = Q(\sqrt{2x})$, where $Q(\cdot)$ is the Gaussian-Q function, which is defined as $Q(x) = (1/\sqrt{2\pi}) \int_x^\infty e^{-(t^2/2)} dt$, giving

$$P_e = \frac{1}{\sqrt{4\pi}} \int_0^\infty F_{\gamma_k}^N(x) \frac{e^{-x}}{\sqrt{x}}\, dx \tag{8}$$

because $Q(\sqrt{2x}) F_{\gamma_k}^N(x)_0^\infty = 0$, and $dQ(\sqrt{2x}) = -(1/\sqrt{4\pi})(e^{-x}/\sqrt{x})$. For Rayleigh fading channels, $m_1 = m_2 = 1$, and thus, (8) is specialized to

$$P_e = \frac{1}{\sqrt{4\pi}} \int_0^\infty \left[1 - 2e^{-\left(\frac{1}{\bar\gamma_1} + \frac{1}{\bar\gamma_2}\right)x} \sqrt{x(x+1)/\bar\gamma_1/\bar\gamma_2} \cdot K_{-1}\left(2\sqrt{x(1+x)/\bar\gamma_1/\bar\gamma_2}\right)\right]^N \frac{e^{-x}}{\sqrt{x}}\, dx \tag{9}$$

by replacing m_1 and m_2 in (5) with 1 and using the replaced expression of $F_{\gamma_k}^N(x)$ in (8). When x is large, $K_{-1}(x) \approx (1/x)$. Using this, (9) can be approximated as

$$P_e \approx \frac{1}{2} + \frac{1}{2} \sum_{i=1}^N \frac{\binom{N}{i}(-1)^i}{\sqrt{1 + i/\bar\gamma_1 + i/\bar\gamma_2}}. \tag{10}$$

Equations (8) and (9) can be numerically calculated with only one single integral, whereas (10) is calculated in closed form. All of them are new results that are not available in the literature. They will be used as benchmarks to compare different partial selection schemes.

B. Conventional Partial Selection Scheme

In [12], the conventional partial selection scheme chooses the idle user according to

$$K = \max_{k=1,2,\ldots,N} \{\gamma_{1,k}\} \tag{11}$$

by using the instantaneous SNR of the first hop only. Denote this scheme as the $\max\{\gamma_{1,k}\}$ scheme. The $\max\{\gamma_{1,k}\}$ scheme greatly reduces the complexity of cooperative diversity. The results in [12] are for Rayleigh fading channels. We extend them to Nakagami-m fading channels. Similar to (4), the probability of error in this case is given by $P_e = \int_0^\infty P(e|x) dF_{\gamma_K}(x)$, where $P(e|x)$ is again the conditional probability of error, which is conditioned on the instantaneous end-to-end SNR of the chosen link γ_K, and $F_{\gamma_K}(x)$ is the cdf of γ_K derived in the Appendix as

$$\begin{aligned} F_{\gamma_K}(x) = 1 + &\sum_{i=1}^N \sum_{j_i=0}^{m_1-1} \sum_{l_1=0}^{m_2-1} \sum_{l_2=0}^{j_1+\cdots+j_i} 2(-1)^i m_2^{m_2} \binom{N}{i} \\ &\frac{\binom{m_2-1}{l_1}\binom{j_1+\cdots+j_i}{l_2}[(m_1-1)!]^i}{\Gamma^i(m_1)\bar\gamma_2^{m_2}\Gamma(m_2) j_1!\cdots j_i!} \\ &\cdot \left(\frac{m_2}{\bar\gamma_2}\right)^{\frac{l_2-l_1-1}{2}} \left(\frac{m_1}{\bar\gamma_1}\right)^{\frac{l_1-l_2+1}{2}+j_1+\cdots+j_i} \\ &i^{\frac{l_1-l_2+1}{2}} x^{j_1+\cdots+j_i+m_2-1-l_1+\frac{l_1-l_2+1}{2}} \\ &\cdot (x+1)^{\frac{l_1+l_2+1}{2}} e^{-\left(\frac{im_1}{\bar\gamma_1}+\frac{m_2}{\bar\gamma_2}\right)x} \\ &\times K_{l_1-l_2+1}\left(2\sqrt{\frac{im_1m_2x(x+1)}{\bar\gamma_1\bar\gamma_2}}\right). \end{aligned} \tag{12}$$

Similarly, using integration by parts, one has

$$P_e = P(e|x)F_{\gamma_K}(x)_0^\infty - \int_0^\infty F_{\gamma_K}(x) dP(e|x). \tag{13}$$

For BPSK, one has $P(e|x) = Q(\sqrt{2x})$, $Q(\sqrt{2x})F_{\gamma_K}(x)_0^\infty = 0$ and $dQ(\sqrt{2x}) = -(1/\sqrt{4\pi})(e^{-x}/\sqrt{x})$. Then, the error rate in (13) can be calculated as

$$P_e = \frac{1}{\sqrt{4\pi}} \int_0^\infty F_{\gamma_K}(x) \frac{e^{-x}}{\sqrt{x}}\, dx \tag{14}$$

where $F_{\gamma_K}(x)$ is given by (12). Again, in Rayleigh fading channels, by replacing m_1 and m_2 with 1 in (12) and using the replaced expression in (14), (14) is specialized to

$$P_e = \frac{1}{\sqrt{4\pi}} \int_0^\infty \left[1 + \frac{1}{\bar\gamma_2} \sum_{i=1}^N \binom{N}{i}(-1)^i e^{-\frac{ix}{\bar\gamma_1} - \frac{x}{\bar\gamma_2}} \sqrt{\frac{4\bar\gamma_2 ix(x+1)}{\bar\gamma_1}} K_1\left(2\sqrt{\frac{i(x+1)x}{\bar\gamma_1\bar\gamma_2}}\right)\right] \frac{e^{-x}}{\sqrt{x}}\, dx. \tag{15}$$

Using $K_1(x) = K_{-1}(x)$ and $K_{-1}(x) \approx (1/x)$ for large x, one can also approximate (15) as

$$P_e \approx \frac{1}{2} + \frac{1}{2} \sum_{i=1}^N \frac{\binom{N}{i}(-1)^i}{\sqrt{1 + \frac{i}{\bar\gamma_1} + \frac{1}{\bar\gamma_2}}}. \tag{16}$$

It can be verified that (16) agrees with [12, eq. (14)]. Equations (14) and (15) are also new results that are not available in the literature. They will be used to compare with the new schemes.

C. New Partial Selection Schemes

In the conventional partial selection scheme, one chooses the idle user with the strongest first-hop instantaneous SNR. One can also derive a new partial selection scheme that chooses the idle user with the strongest second-hop instantaneous SNR as

$$K = \max_{k=1,2,\ldots,N} \{\gamma_{2,k}\}. \tag{17}$$

Denote this scheme as the $\max\{\gamma_{2,k}\}$ scheme. Note that the $\max\{\gamma_{2,k}\}$ scheme chooses the link with the strongest second-hop, which is not equivalent to choosing the second strongest link. Similarly, in Nakagami-m fading channels, its probability of error for BPSK is (14), except that the cdf of γ_K is derived in the Appendix as

$$\begin{aligned} F_{\gamma_K}(x) = 1 + &\sum_{i=1}^N \sum_{j_i=0}^{m_2-1} \sum_{l_1=0}^{m_1-1} \sum_{l_2=0}^{j_1+\cdots+j_i} 2(-1)^i m_1^{m_1} \binom{N}{i} \\ &\times \frac{\binom{m_1-1}{l_1}\binom{j_1+\cdots+j_i}{l_2}[(m_2-1)!]^i}{\Gamma^i(m_2)\bar\gamma_1^{m_1}\Gamma(m_1) j_1!\cdots j_i!} \\ &\cdot \left(\frac{m_1}{\bar\gamma_1}\right)^{\frac{l_2-l_1-1}{2}} \left(\frac{m_2}{\bar\gamma_2}\right)^{\frac{l_1-l_2+1}{2}+j_1+\cdots+j_i} \\ &\times i^{\frac{l_1-l_2+1}{2}} x^{j_1+\cdots+j_i+m_1-1-l_1+\frac{l_1-l_2+1}{2}} \\ &\cdot (x+1)^{\frac{l_1+l_2+1}{2}} e^{-\left(\frac{im_2}{\bar\gamma_2}+\frac{m_1}{\bar\gamma_1}\right)x} \end{aligned}$$

$$\times K_{l_1-l_2+1}\left(2\sqrt{\frac{im_1m_2x(x+1)}{\bar{\gamma}_1\bar{\gamma}_2}}\right). \quad (18)$$

In Rayleigh fading channels, the probability of error for BPSK is given by

$$P_e = \frac{1}{\sqrt{4\pi}}\int_0^\infty \left[1+\frac{1}{\bar{\gamma}_1}\sum_{i=1}^{N}\binom{N}{i}(-1)^i e^{-\frac{ix}{\bar{\gamma}_2}-\frac{x}{\bar{\gamma}_1}} \times\sqrt{\frac{4\bar{\gamma}_1 ix(x+1)}{\bar{\gamma}_2}}K_1\left(2\sqrt{\frac{i(x+1)x}{\bar{\gamma}_1\bar{\gamma}_2}}\right)\right]\frac{e^{-x}}{\sqrt{x}}\,dx \quad (19)$$

which can be approximated as

$$P_e \approx \frac{1}{2}+\frac{1}{2}\sum_{i=1}^{N}\frac{\binom{N}{i}(-1)^i}{\sqrt{1+\frac{1}{\bar{\gamma}_1}+\frac{i}{\bar{\gamma}_2}}}. \quad (20)$$

The aforementioned selection schemes use the instantaneous SNRs for selection. This requires channel estimators for $h_{1,k}$ or $h_{2,k}$ or both, $k = 1, 2, \ldots, N$. It is also effective to use the received signal amplitude for selection [18]. Thus, two new partial selection schemes are

$$K = \max_{k=1,2,\ldots,N}\{|u_k|\} \quad (21)$$

$$K = \max_{k=1,2,\ldots,N}\{|y_k|\}. \quad (22)$$

Denote (21) as the $\max\{|u_k|\}$ scheme and (22) as the $\max\{|y_k|\}$ scheme. The selection of the idle user is made at the base station in a centralized network or at the group leader in a distributed network. The decision will be broadcast by the base station or the group leader to the source, the destination, and the idle users. The implementation details are not shown as they are beyond the scope of this paper.

Assume that each real symbol transmission costs the same overhead P and each channel estimation uses Q real symbols. The $\max\{|u_k|\}$ scheme requires transmission of the N received real amplitudes at the idle users for selection and $h_{1,K}$ and $h_{2,K}$ for demodulation. The overhead costs $NP + 2QP$. The $\max\{|y_k|\}$ scheme requires transmission of the N received real amplitudes at the destination for selection, which requires N channel estimators from $h_{1,1}$ to $h_{1,N}$ to calculate the amplification factors for forwarding the signals and $h_{2,K}$ for demodulation. The overhead costs $NP + (N+1)QP$. The $\max\{\gamma_{2,k}\}$ scheme requires transmission of the N complex channel estimates from $h_{2,1}$ to $h_{2,N}$ for selection and $h_{1,K}$ for demodulation. The overhead costs $2NP + (N+1)QP$. The conventional $\max\{\gamma_{1,k}\}$ scheme requires transmission of the N complex channel estimates from $h_{1,1}$ to $h_{1,N}$ for selection and $h_{2,K}$ for demodulation. The overhead costs $2NP + (N+1)QP$. The full selection scheme requires transmission of N complex channel estimates from $h_{1,1}$ to $h_{1,N}$ and N complex channel estimates from $h_{2,1}$ to $h_{2,N}$ for selection and demodulation. The overhead costs $4NP + 2NQP$. Thus, the partial selection schemes are simpler than the full selection scheme, the new $\max\{|u_k|\}$ and $\max\{|y_k|\}$ schemes are simpler than the conventional $\max\{\gamma_{1,k}\}$ scheme, whereas the new $\max\{\gamma_{2,k}\}$ scheme has the same complexity as the conventional $\max\{\gamma_{1,k}\}$ scheme. Among the new schemes, the $\max\{|u_k|\}$ scheme is simplest, the $\max\{|y_k|\}$ scheme is second simplest, and the $\max\{\gamma_{2,k}\}$ scheme is most complicated. The complexity reduction increases when N increases. Derivation of the error rate for amplitude-based selection in (21) and (22) has been a long-standing problem and is not available [19].

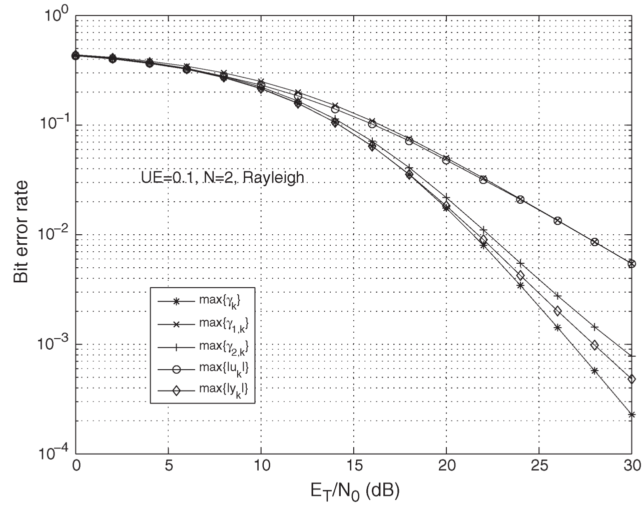

Fig. 1. Comparison of different partial selection schemes at $UE = 0.1$ and $N = 2$ in Rayleigh fading channels.

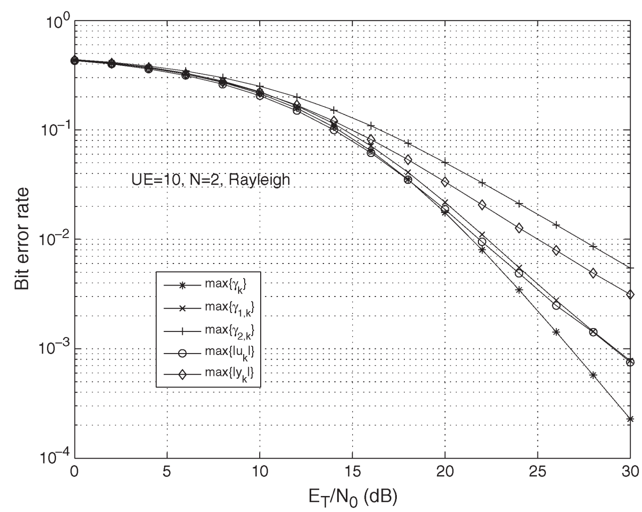

Fig. 2. Comparison of different partial selection schemes at $UE = 10$ and $N = 2$ in Rayleigh fading channels.

III. Numerical Results and Discussion

Here, numerical examples are presented to compare the performances of different partial selection schemes. In the comparison, BPSK is used. In addition, $\Omega_1 = \Omega_2 = 1$, and $N_1 = N_2 = N_0 = 1$, whereas $E_1 = E_T/N(UE+1)$, and $E_2 = E_T * UE/N(UE+1)$, where $*$ represents product, $E_T = (E_1 + E_2)N$ is the total energy, and $UE = E_2/E_1$ is the ratio of E_2 to E_1. Since $\Omega_1 = \Omega_2$ and $N_1 = N_2$, UE is also the ratio of $\bar{\gamma}_2$ to $\bar{\gamma}_1$.

Fig. 1 compares different partial selection schemes when $N = 2$ and $UE = 0.1$ in Rayleigh fading channels. One sees that all the new partial selection schemes outperform the conventional $\max\{\gamma_{1,k}\}$ scheme. For example, at a bit error rate of 10^{-2}, the new $\max\{\gamma_{2,k}\}$ and $\max\{|y_k|\}$ schemes have performance gains of around 5 dB over the conventional $\max\{\gamma_{1,k}\}$ scheme. Comparing the new partial selection schemes, one sees that the $\max\{|y_k|\}$ scheme performs the best. Its performance is indistinguishable from the performance of the full selection $\max\{\gamma_k\}$ scheme when the SNR is less than 20 dB. Fig. 2 compares different partial selection schemes when

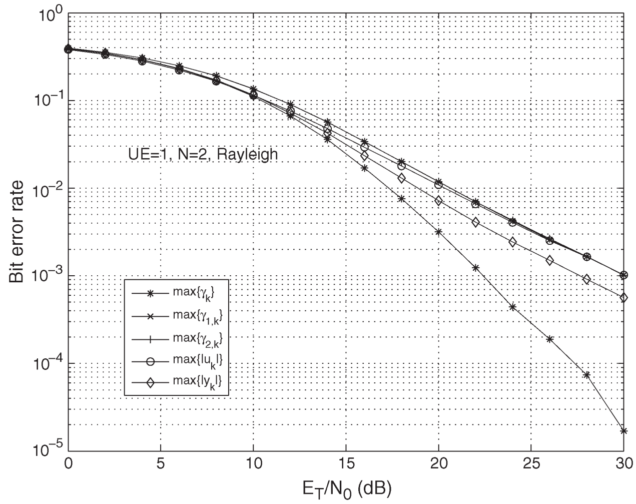

Fig. 3. Comparison of different partial selection schemes at $UE = 1$ and $N = 2$ in Rayleigh fading channels.

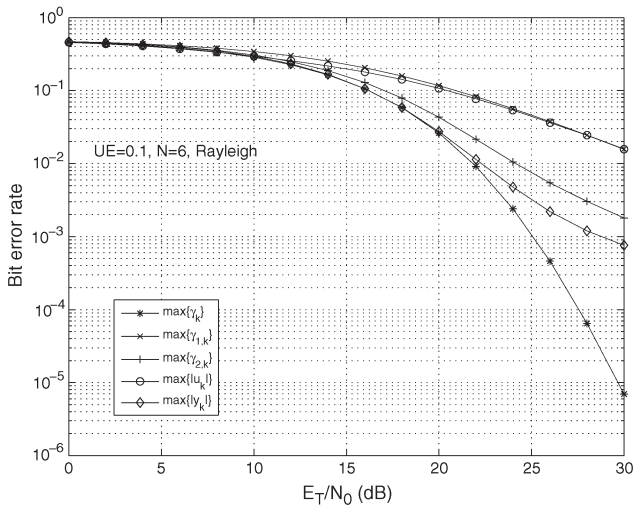

Fig. 4. Comparison of different partial selection schemes at $UE = 0.1$ and $N = 6$ in Rayleigh fading channels.

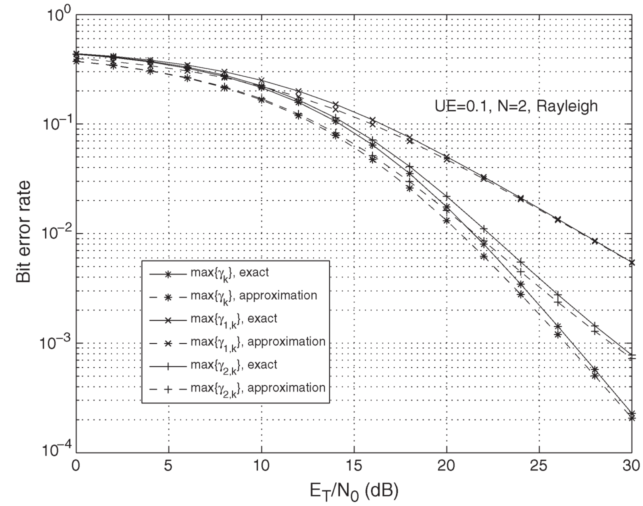

Fig. 5. Comparison of the exact error rates and the approximate error rates for different partial selection schemes at $UE = 0.1$ and $N = 2$ in Rayleigh fading channels.

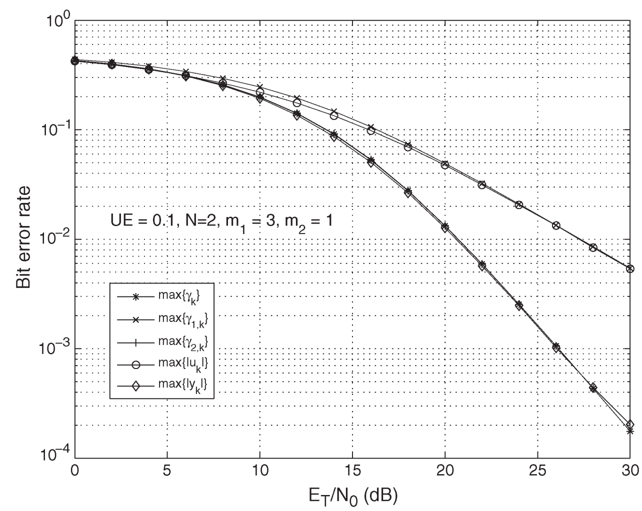

Fig. 6. Comparison of different partial selection schemes at $UE = 0.1$, $N = 2$, $m_1 = 3$, and $m_2 = 1$ in Nakagami-m fading channels.

$N = 2$ and $UE = 10$ in Rayleigh fading channels. In this case, the new $\max\{|u_k|\}$ scheme still outperforms the conventional $\max\{\gamma_{1,k}\}$ scheme. Among the new schemes, the $\max\{|u_k|\}$ scheme performs the best. Since the second hop has a smaller average SNR ($\bar{\gamma}_2 = 0.1\bar{\gamma}_1$) in Fig. 1 and the first hop has a smaller average SNR ($\bar{\gamma}_2 = 10\bar{\gamma}_1$) in Fig. 2, one concludes that one should choose the best idle user for the hop with a smaller average SNR to achieve maximum bit error rate performance in partial selection. This may be explained as follows: The value of γ_k approaches $\gamma_{1,k}$ when $\gamma_{2,k}$ is large, and it approaches $\gamma_{2,k}$ when $\gamma_{1,k}$ is large. Thus, it is necessary to make choices in the weaker hop on average. Fig. 3 compares different partial selection schemes when $N = 2$ and $UE = 1$ in Rayleigh fading channels. In this case, both hops have the same average SNR. One sees that both the new $\max\{|u_k|\}$ and $\max\{|y_k|\}$ schemes outperform the conventional $\max\{\gamma_{1,k}\}$ scheme, whereas the performance of the new $\max\{\gamma_{2,k}\}$ scheme is graphically indistinguishable from the performance of the conventional $\max\{\gamma_{1,k}\}$ scheme. In addition, the $\max\{|y_k|\}$ scheme performs the best among the new schemes.

Fig. 4 compares different schemes when $N = 6$ and $UE = 0.1$ in Rayleigh fading channels. Similar observations to those from Fig. 1 can be made. In addition, the performance of the full selection scheme improves when N increases, whereas the performances of the partial selection schemes do not. This is due to the fact that the full selection scheme has a diversity order of N, whereas the partial selection scheme has a diversity order of only 1 to achieve lower complexity. Fig. 5 compares the exact performances of the $\max\{\gamma_k\}$, $\max\{\gamma_{1,k}\}$, and $\max\{\gamma_{2,k}\}$ schemes with their approximate performances in (10), (16), and (20), respectively. One sees that the approximation error decreases when the SNR increases. Therefore, the approximations in (10), (16), and (20) can be used to predict the asymptotic performances, which are defined as the system performances when the SNR approaches infinity. Fig. 6 compares different schemes in Nakagami-m fading channels at $UE = 0.1$, $m_1 = 3$, and $m_2 = 1$. In this example, all the new partial selection schemes outperform the conventional $\max\{\gamma_{1,k}\}$ scheme. In particular, the performances of the $\max\{\gamma_{2,k}\}$ and $\max\{|y_k|\}$ schemes are almost identical to that of the full selection scheme. This agrees with previous observations from Figs. 1 and 2 that the best idle user for the hop with a smaller average SNR should be chosen as $\bar{\gamma}_2 = 0.1\bar{\gamma}_1$. However, when $\bar{\gamma}_2 = 0.1\bar{\gamma}_1$ but $m_2 > m_1$, as shown in Fig. 7, one sees that the best idle user for the hop with a smaller m parameter should be chosen at large SNRs, as the $\max\{\gamma_{1,k}\}$ and $\max\{|u_k|\}$ schemes outperform the $\max\{\gamma_{2,k}\}$ and $\max\{|y_k|\}$ schemes at large SNRs, despite the fact that $\bar{\gamma}_2 = 0.1\bar{\gamma}_1$. One also notes that the $\max\{|y_k|\}$

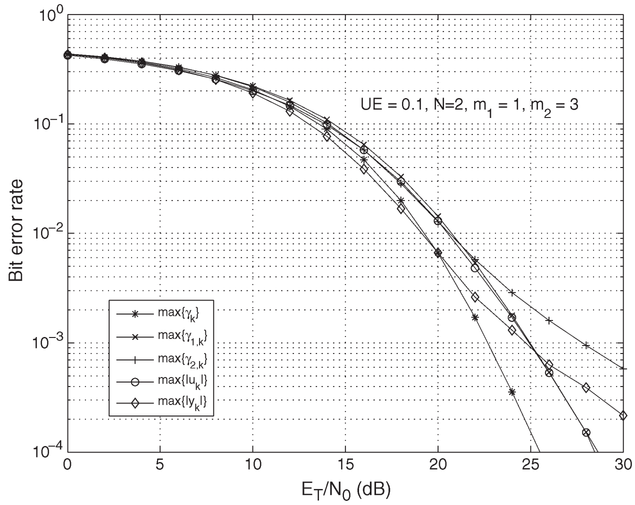

Fig. 7. Comparison of different partial selection schemes at $UE = 0.1$, $N = 2$, $m_1 = 1$, and $m_2 = 3$ in Nakagami-m fading channels.

scheme outperforms the $\max\{\gamma_k\}$ scheme for small SNRs. This was also observed in [18], where selection based on amplitude outperforms that based on SNR. It has been explained in [18] that the orientation of the noise vector may improve the performance of amplitude-based selection.

One concludes from Figs. 1–7 that, if the m parameters are the same for both hops, the best idle user for the hop with a smaller average SNR should be chosen. On the other hand, if the m parameters are different for the two hops, the best idle user for the hop with a smaller average SNR should be chosen at small SNRs and that with a smaller m parameter should be chosen at large SNRs. This observation motivates a new adaptive partial selection scheme by choosing the instantaneous SNR or the received signal amplitude of either the first hop or the second hop according to their average SNRs and m parameters. This scheme outperforms schemes using either the first hop or the second hop alone, at the cost of extra knowledge of the average SNR and the m parameter, which can be accurately estimated using [15].

Appendix
Derivation of (12) and (18)

In the $\max\{\gamma_{1,k}\}$ scheme, one has the instantaneous end-to-end SNR of the chosen link as

$$\gamma_K = \frac{\gamma_{1,K}\gamma_{2,K}}{\gamma_{1,K} + \gamma_{2,K} + 1} \tag{23}$$

where $\gamma_{1,K} = \max_{k=1,2,\ldots,N}\{\gamma_{1,k}\}$, and $\gamma_{2,K}$ is the instantaneous SNR in the second hop of the chosen link. The cdf of $\gamma_{1,K}$ can be derived as $F_{\gamma_{1,K}}(x) = [1 - (\Gamma(m_1, m_1 x/\bar{\gamma}_1)/\Gamma(m_1))]^N$, where $\Gamma(\cdot,\cdot)$ is the incomplete Gamma function [17, eq. (8.350.2)]. The pdf of $\gamma_{2,K}$ is $f_{\gamma_{2,K}}(x) = (m_2^{m_2} x^{m_2-1}/\bar{\gamma}_2^{m_2}\Gamma(m_2))e^{-(m_2/\bar{\gamma}_2)x}$. Following similar methods in [16], the cdf of γ_K is

$$\begin{aligned} F_{\gamma_K}(x) &= \int_0^\infty Pr\left\{\frac{x_1 x_2}{x_1 + x_2 + 1} \le x | x_2\right\} f_{\gamma_{2,K}}(x_2)\, dx_2 \\ &= \int_0^x f_{\gamma_{2,K}}(x_2)\, dx_2 \\ &\quad + \int_x^\infty F_{\gamma_{1,K}}\left(\frac{x(x_2+1)}{x_2 - x}\right) f_{\gamma_{2,K}}(x_2)\, dx_2 \\ &= 1 + \sum_{i=1}^N \frac{(-1)^i m_2^{m_2}\binom{N}{i}}{\Gamma^i(m_1)\bar{\gamma}_2^{m_2}\Gamma(m_2)} \\ &\quad \times \int_x^\infty \Gamma^i\left(m_1, \frac{m_1 x(x_2+1)}{\bar{\gamma}_1(x_2 - x)}\right) x_2^{m_2-1} e^{-\frac{m_2}{\bar{\gamma}_2}x_2}\, dx_2. \end{aligned} \tag{24}$$

Using [17, eq. (8.352.2)] and [17, eq. (3.471.9)] and after some mathematical manipulations, one has (12). When the $\max\{\gamma_{2,k}\}$ scheme is used, the instantaneous end-to-end SNR of the chosen link is

$$\gamma_K = \frac{\gamma_{1,K}\gamma_{2,K}}{\gamma_{1,K} + \gamma_{2,K} + 1} \tag{25}$$

where $\gamma_{2,K} = \max_{k=1,2,\ldots,N}\{\gamma_{2,k}\}$, and $\gamma_{1,K}$ is the instantaneous SNR in the first hop of the chosen link. Due to symmetry, one can obtain (18).

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Hybrid Tree Search Algorithms for Detection in Spatial Multiplexing Systems

Kuei-Chiang Lai, Jiun-Jie Jia, and Li-Wei Lin

***Abstract*—Hybrid tree search algorithms are described for maximum-likelihood symbol detection in spatial multiplexing (SM) systems. Essentially, the search tree is iteratively expanded in breadth-first (BF) manner until the probability that the current most likely path is correct exceeds the specified threshold, at which point, the depth-first (DF) stage is initiated to traverse the rest of the tree. In contrast with the sphere decoding (SD) algorithm, which starts off with the DF search, the proposed algorithms use the BF stage to enhance the accuracy of the initial DF search direction by exploiting the diversity inherent in the SM scheme. Simulation results demonstrate that, with a moderate increase in the memory requirement, the proposed algorithms achieve a significantly lower complexity than the SD algorithm in many scenarios.**

***Index Terms*—Mulitple-input–multiple-output (MIMO), sphere decoding, tree search.**

I. Introduction

Multiple-input–multiple-output (MIMO) technology is considered by many to be one of the most promising means to provide high data rates and reliable link quality for wireless communications systems [1]. The spatial multiplexing (SM) scheme is a MIMO transmission technique that can boost the system capacity [1]. In such a scheme, independent data streams are simultaneously sent over different transmit (Tx) antennas. These streams interfere with each other at receive (Rx) antennas, which calls for advanced detection algorithms for successful separation.

Assuming that transmitted symbols are equiprobable, the maximum-likelihood (ML) detector minimizes the probability of detection errors for the SM scheme; however, the complexity of brute-force implementation (which searches over all the hypotheses) is too high for practical use. ML detection algorithms using tree search (see [2] for a review) achieve significant complexity saving and have thus recently received much attention. The search strategies [2] include the breadth-first (BF) search (e.g., the BF signal decoder [3]), the best-first search (e.g., the stack algorithm [2]), and the depth-first (DF) search (e.g., the sphere decoding (SD) algorithm [4]). Both the BF and best-first approaches have a worst-case memory requirement that exponentially grows with the number of Tx antennas and are therefore not suitable for hardware implementation in a *large MIMO system* (i.e., when a large number of Tx antennas and high-order modulation are used). However, the stack algorithm is the most efficient, *in terms of the number of tree nodes expanded* (a rough estimate of the complexity), among all the ML tree search algorithms [2]. In contrast, the SD algorithm has a memory requirement that linearly grows with the number of Tx antennas, but it expands many more nodes than the stack algorithm in a large MIMO system. From the preceding discussion, ML tree search algorithms achieving a better tradeoff between the complexity and memory usage are highly desirable, which is the problem addressed in this paper.

In [5], memory-constrained tree search (MCTS), which modifies the stack algorithm to reduce the memory requirement while maintaining ML performance, is proposed. The amount of memory allocated dictates whether MCTS behaves more like the (best-first) stack algorithm or the (DF) SD algorithm, thus providing a flexible tradeoff between the complexity and memory usage. In this paper, we take a different approach by combining the BF search with the SD algorithm to improve the accuracy of the initial DF search direction. The goal is to reduce the average and worst-case complexity of the SD algorithm (via the BF stage) while maintaining a memory requirement that linearly grows with the number of Tx antennas (via the DF stage). Simulation results show that the proposed algorithms achieve a better tradeoff between the complexity and memory usage than the SD algorithm and MCTS in many scenarios. This paper is a journal version of [6]. The extension includes sort-free implementation, analysis of memory requirements, a suboptimal algorithm that achieves a near-ML performance with further complexity reduction, and more comprehensive comparisons in performance, complexity, and memory requirements.

The rest of this paper is organized as follows: Section II describes the signal model. Section III introduces the formation and representation of the search tree and reviews the SD algorithm [4], which serves as the baseline for algorithm development and complexity comparison. The proposed algorithms and the simulation results are discussed in Sections IV and V, respectively.

II. Signal Model

Consider an $n_t \times n_r$ SM system in which the Tx and Rx sites are equipped with n_t and n_r antennas, respectively, with $n_r \geq n_t$. At any time instant, the received signal samples at the Rx antennas can be stacked into a vector $\mathbf{y}$ whose ith element y_i is associated with the ith Rx antenna. Assuming a flat-fading channel between any pair of Tx and Rx antennas, the received signal vector can be modeled as $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$, where $\mathbf{H}$ is the channel matrix, $\mathbf{x}$ is the transmitted symbol vector at that time instant, and $\mathbf{n}$ is the noise vector. Note that the (i, j)th element of $\mathbf{H}$, which is denoted as h_{ij}, is the gain of the scalar channel between the jth Tx antenna and the ith Rx antenna. The ith entry of $\mathbf{x}$, which is denoted as x_i, is the modulation symbol (taken from a constellation C of size $|C|$) transmitted from the ith Tx antenna. It is assumed that the elements of $\mathbf{n}$ are independent and identically distributed (i.i.d.) complex Gaussian random variables (RVs), each having a zero mean and a variance of σ_n^2, which are denoted as $\mathcal{CN}(0, \sigma_n^2)$.

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The authors are with the Department of Electrical Engineering, Institute of Computer and Communication Engineering, National Cheng Kung University, Tainan 70101, Taiwan (e-mail: kclai@mail.ncku.edu.tw; chia1112@hotmail.com; demon0821@hotmail.com).

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