Level-Crossing Rate and Average Duration of Fades of Deterministic Simulation Models for Nakagami-Hoyt Fading Channels

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Abstract

In this paper, an efficient deterministic simulation model for the Nakagami-Hoyt fading channel model (Q model) is proposed by using the concept of Rice's sum of sinusoids. Analytical formulas are derived for the amplitude and phase probability density functions (PDF), level-crossing rate (LCR) and average duration of fades (ADF). By using a numerical optimization procedure, we show how the statistical properties of the Q model and the corresponding simulation model can be adapted to those of an equivalent mobile satellite channel for an environment with heavy shadowing. It is demonstrated by several theoretical and simulation results that the Q model and the therefrom derived simulation model can provide excellent characterization for the corresponding measurement results with respect to the complementary cumulative distribution function (CDF), normalized LCR and ADF. Finally, it is shown that the Q model enables a better statistical fitting to the measurement data compared to the Rayleigh model due to the increased flexibility.

Keywords

Nakagami-Hoyt channel model (Q model), deterministic channel modeling, level-crossing rate, average duration of fades.

INTRODUCTION

The statistics of fading time intervals are of great importance for the design, development, and performance analysis of mobile radio communication systems. Specifically, the LCR, which provides us with a measure of the average number of crossings per second at which the envelope crosses a specified signal level with positive slope, is an important second order statistical quantity that characterizes the rate of occurrence of fades. The ADF, which is another second order statistical quantity, enables one to get the statistics of burst errors occurring in fading channels [1, 2], and thus, can provide useful information for the design of error-correcting schemes. Furthermore, the above described statistics are also important for optimizing the interleaver size [3], analyzing the system throughput [4], as well as for performance studies of transmission schemes in general [5].

In the crossing theory, it is well known that the analytical solutions for the LCR and ADF depend strictly on the amplitude distribution of the considered process, i.e., the fading channel model. Closed-form expressions for these quantities have been provided, e.g., for Rayleigh [6], Rice [7], lognormal [8], Suzuki [9], modified Suzuki [9], and Nakagami channel models [10]. Recently, analytical expressions for the LCR and ADF in the case of the Q model, the amplitude distribution of which is given by the Q distribution [11, 12, 13], have been derived in [14]. The Q distribution allows us to span the range of fading distribution from one-sided Gaussian fading to Rayleigh fading and approximates, in certain amplitude range, the Nakagami distribution [12]. In [15], the Q distribution has been shown to serve as a complement distribution for a new mobile satellite propagation channel model which is a combination of a good channel (Rice distribution) and a bad channel (Q distribution). Therefore, the results obtained in [14] are useful, together with the known statistics of the Rice model, for investigating the crossing statistics of the combination model presented in [15].

An efficient channel simulator, which can reproduce the required statistics of the reference model, is essential for the design, optimization and test of modern wireless communication systems. In this paper, a deterministic simulation model for the Q model is proposed. Analytical expressions for the PDF of the amplitude and phase, LCR and ADF of the simulation model are derived. Furthermore, the usefulness of the Q model and the corresponding simulation model is demonstrated by fitting the analytical statistics to experimental measurement data.

THE Q MODEL AND ITS STATISTICS

In this section, we briefly review the statistics of Q fading channels. Throughout the paper, we will make use of the complex baseband notation.

A Q process, $R(t)$, is obtained by taking the absolute value of a complex Gaussian random process

$$\mu(t) = \mu_1(t) + j\mu_2(t) \tag{1}$$

according to

$$R(t) = |\mu(t)| \, . \tag{2}$$

In (1), $\mu_1(t)$ and $\mu_2(t)$ are uncorrelated real Gaussian noise processes with zero mean and variances σ_1^2 and σ_2^2, respectively. The amplitude PDF of $R(t)$ is given by [11]

$$p_R(x) = \frac{x}{\sigma_1\sigma_2}\exp[-\frac{x^2}{4}(\frac{1}{\sigma_1^2}+\frac{1}{\sigma_2^2})] \cdot I_0[\frac{x^2}{4}(\frac{1}{\sigma_2^2}-\frac{1}{\sigma_1^2})], \quad x \geq 0, \quad (3)$$

where $I_0(\cdot)$ denotes the zeroth-order modified Bessel function of the first kind. The corresponding phase process, $\vartheta(t) = \arctan[\mu_2(t)/\mu_1(t)]$, has the following PDF [11]

$$p_\vartheta(\theta) = \frac{\sigma_1\sigma_2}{2\pi(\sigma_2^2\cos^2\theta + \sigma_1^2\sin^2\theta)}, \quad 0 \leq \theta < 2\pi. \quad (4)$$

It should be mentioned that the expressions (3) and (4) reduce to the Rayleigh PDF and the uniform PDF [6], respectively, if $\mu_1(t)$ and $\mu_2(t)$ have equal variances, i.e., $\sigma_1^2 = \sigma_2^2$.

Our further interest is the LCR and ADF of the Q process $R(t)$, which will be denoted here by $N_R(r)$ and $T_{R-}(r)$, respectively. It has been shown in [14] that the LCR $N_R(r)$ can be expressed by

$$N_R(r) = \frac{r}{(2\pi)^{3/2}\sigma_1\sigma_2} \cdot \int_0^{2\pi} \exp[-\frac{r^2}{2\sigma_1^2\sigma_2^2}(\sigma_2^2\cos^2\theta + \sigma_1^2\sin^2\theta)] \cdot \sqrt{\beta_2\sin^2\theta + \beta_1\cos^2\theta}\, d\theta, \quad (5)$$

where $\beta_i = -\ddot{r}_{\mu_i\mu_i}(0)$ $(i = 1, 2)$ is related to the curvature of the auto-correlation function (ACF) $r_{\mu_i\mu_i}(\tau)$ of $\mu_i(t)$ at $\tau = 0$. On condition that $\sigma_1^2 = \sigma_2^2 = \sigma^2$ $(\beta_1 = \beta_2 = \beta)$, we can easily show that (5) reduces to the LCR of Rayleigh processes [6], i.e.,

$$N_R(r) = \sqrt{\frac{\beta}{2\pi}}\frac{r}{\sigma^2}\exp(-\frac{r^2}{2\sigma^2}). \quad (6)$$

The ADF $T_{R-}(r)$ is the mean value of the time intervals over which the process $R(t)$ is below a specified level r. Generally, the ADF $T_{R-}(r)$ is related to the LCR $N_R(r)$ as follows [6, 7]

$$T_{R-}(r) = \frac{P_{R-}(r)}{N_R(r)}, \quad (7)$$

where $P_{R-}(r)$ is the CDF of the Q process $R(t)$, which is defined by

$$P_{R-}(r) = \int_0^r p_R(x)dx. \quad (8)$$

Note that the ADF $T_{R-}(r)$ can immediately be computed by substituting (3), (5) and (8) into (7).

A DETERMINISTIC SIMULATION MODEL

In this section, an efficient deterministic simulation model based on Rice's sum of sinusoids [16, 17] is presented and its statistical properties are investigated. The statistics of the underlying real Gaussian noise processes $\mu_i(t)$ $(i = 1, 2)$ of the Q model are approximated by using the following sum of sinusoids

$$\tilde{\mu}_i(t) = \sum_{n=1}^{N_i} c_{i,n}\cos(2\pi f_{i,n}t + \Phi_{i,n}), \quad (9)$$

where N_i designates the number of sinusoids. In (9), $c_{i,n}$, $f_{i,n}$ and $\Phi_{i,n}$ are called the Doppler coefficients, discrete Doppler frequencies and Doppler phases, respectively. These simulation model parameters have to be determined in such a way that the statistics of the simulation model approximate those of the Q model as close as possible. In this paper, the so-called Method of Exact Doppler Spread (MEDS) [18] is applied for the computation of these quantities. It's worth mentioning that all these parameters determined by the MEDS are kept constant during simulation. Consequently, $\tilde{\mu}_i(t)$ is a deterministic function and the resulting simulator is of deterministic nature. In general, the ACF of $\tilde{\mu}_i(t)$ $(i = 1, 2)$ is given by [18]

$$\tilde{r}_{\mu_i\mu_i}(\tau) = \sum_{n=1}^{N_i}\frac{c_{i,n}^2}{2}\cos(2\pi f_{i,n}\tau). \quad (10)$$

In this paper, we will restrict the application of the MEDS to the Jakes Doppler power spectral density [6]. In this case, the corresponding ACF of $\mu_i(t)$ $(i = 1, 2)$ is given by

$$r_{\mu_i\mu_i}(\tau) = \sigma_i^2 J_0(2\pi f_{max_i}\tau), \quad (11)$$

where $J_0(\cdot)$ denotes the zeroth-order Bessel function of the first kind and f_{max_i} represents the maximum Doppler frequency of $\mu_i(t)$. In (5), the quantity β_i $(i = 1, 2)$ can then be obtained from (11) as

$$\beta_i = -\ddot{r}_{\mu_i\mu_i}(0) = 2(\pi\sigma_i f_{max_i})^2. \quad (12)$$

According to the MEDS, $c_{i,n}$ and $f_{i,n}$ are given by

$$c_{i,n} = \sigma_i\sqrt{\frac{2}{N_i}}, \quad (13)$$

and

$$f_{i,n} = f_{max_i}\sin\left[\frac{\pi}{2N_i}\left(n - \frac{1}{2}\right)\right], \quad (14)$$

respectively. In (14),

$$f_{max_2} = \frac{\sigma_1}{\sigma_2}\sqrt{\frac{\beta_2}{\beta_1}}f_{max}, \quad (15)$$

where $f_{max} = f_{max_1}$. Note that the deterministic processes $\tilde{\mu}_1(t)$ and $\tilde{\mu}_2(t)$ are uncorrelated if and only if $f_{1,n} \neq f_{2,m}$ for all $n = 1, 2, \ldots, N_1$ and $m = 1, 2, \ldots, N_2$ [18]. Therefore, N_1 and N_2 can have the same values under the restriction that $f_{max_1} \neq f_{max_2}$. The Doppler phases $\Phi_{i,n}$ are constant quantities obtained from realizations of a random generator with uniform distribution over the interval $[0, 2\pi)$. It is crucial to note that the described deterministic simulation model for the Q model [see (9)] has essentially the same structure as that of the simulation model for the Rayleigh process [18, 19]. Thus, the analytical expressions for the statistical properties of the simulation model here should also be the same as those of the simulation model for Rayleigh fading channels. Of course, the determined parameters $c_{i,n}$ and $f_{i,n}$ for the Q model and Rayleigh model are different due to the fact that the in-phase and quadrature components of the Q process can have different variances and different maximum Doppler frequencies. Since the first and the second order statistics of deterministic simulation models for Rayleigh processes have been addressed in [18, 19, 20], we only present the related results for shortness in the following. The desired amplitude PDF $\tilde{p}_R(x)$ of $\tilde{R}(t) = |\tilde{\mu}_1(t)+j\tilde{\mu}_2(t)|$ is computed by [18]

$$\tilde{p}_R(x) = x\int_0^{2\pi} \tilde{p}_{\mu_1}(x\cos\theta)\cdot\tilde{p}_{\mu_2}(x\sin\theta)d\theta \ , \qquad (16)$$

where

$$\tilde{p}_{\mu_i}(x_i) = 2\int_0^{\infty}\left[\prod_{n=1}^{N_i} J_0(2\pi c_{i,n}\nu)\right]\cos(2\pi\nu x_i)\, d\nu, \quad i = 1,2\ . \qquad (17)$$

The result of (16) can further be expressed as [19]

$$\tilde{p}_R(x) = 4\pi x\int_0^{\pi}\int_0^{\infty} h_1(z,\theta)h_2(z,\theta)\cdot J_0(2\pi xz)zdzd\theta\ , \qquad (18)$$

where

$$h_1(z,\theta) = \prod_{n=1}^{N_1} J_0(2\pi c_{1,n}z\cos\theta)\ , \qquad (19a)$$

$$h_2(z,\theta) = \prod_{n=1}^{N_2} J_0(2\pi c_{2,n}z\sin\theta)\ . \qquad (19b)$$

Concerning the phase process, its PDF $\tilde{p}_{\vartheta}(\theta)$ is given by [18]

$$\tilde{p}_{\vartheta}(\theta) = \int_0^{\infty} z\tilde{p}_{\mu_1}(z\cos\theta)\cdot\tilde{p}_{\mu_2}(z\sin\theta)dz\ . \qquad (20)$$

The LCR $\tilde{N}_R(r)$ of the deterministic simulation model is expressed as follows [19]

$$\tilde{N}_R(r) = 2r\int_0^{\infty}\int_0^{2\pi}\tilde{p}_{\mu_1}(r\cos\theta)\tilde{p}_{\mu_2}(r\sin\theta) \cdot\int_0^{\infty} j_1(z,\theta)j_2(z,\theta)\dot{z}\cos(2\pi z\dot{z})\, dzd\theta d\dot{z}\ , \qquad (21)$$

where $\dot{z}$ denotes the time derivative of z, and

$$j_1(z,\theta) = \prod_{n=1}^{N_1} J_0(4\pi^2 c_{1,n}f_{1,n}z\cos\theta)\ , \qquad (22a)$$

$$j_2(z,\theta) = \prod_{n=1}^{N_2} J_0(4\pi^2 c_{2,n}f_{2,n}z\sin\theta)\ . \qquad (22b)$$

Finally, the ADF $\tilde{T}_{R-}(r)$ of $\tilde{R}(t)$ can be calculated by [19]

$$\tilde{T}_{R-}(r) = \frac{\tilde{P}_{R-}(r)}{\tilde{N}_R(r)} = \frac{\int_0^r \tilde{p}_R(x)dx}{\tilde{N}_R(r)}\ . \qquad (23)$$

Note that this expression can be evaluated immediately by putting (18) and (21) into (23).

APPLICATIONS AND RESULTS

In this section, we demonstrate the validation of the Q model for describing the statistics of real world mobile fading channels by fitting the analytical statistics to measurement results. Let us therefore consider the measurement results of the complementary CDF $P^*_{R+}(r)$, normalized LCR $N^*_R(r)/f_{max}$ and ADF $T^*_{R-}(r)\cdot f_{max}$ of an equivalent mobile satellite channel for the heavy shadowing environment [21]. In the following, only the measured quantities $P^*_{R+}(r)$ and $N^*_R(r)/f_{max}$ are used as object functions for the optimization of the complementary CDF $P_{R+}(r) = 1 - P_{R-}(r)$ and the normalized LCR $N_R(r)/f_{max}$ of the Q model because the normalized ADF $T_{R-}(r)\cdot f_{max}$ is completely defined by $P_{R-}(r)$ and $N_R(r)/f_{max}$ [see (7)]. Our task now is to find a proper parameter vector $\Omega = (\sigma_1^2, \sigma_2^2, \beta_1, \beta_2)$, which minimizes the difference between the analytical and the measured results. Therefore, we minimize the following error norm

$$E_2(\Omega) = \sqrt{\sum_{m=1}^{M}\left\{W_1(r_m).\left[P_{R+}(r_m) - P^*_{R+}(r_m)\right]\right\}^2} + \sqrt{\sum_{m=1}^{M}\left\{W_2(r_m).\left[\frac{N_R(r_m)}{f_{max}} - \frac{N^*_R(r_m)}{f_{max}}\right]\right\}^2}, \qquad (24)$$

where M is the number of measurement values, $W_1(\cdot)$ and $W_2(\cdot)$ represent appropriate weighting functions, which are simply defined here by scaled versions of the reciprocals of $P^*_{R+}(\cdot)$ and $N^*_R(\cdot)/f_{max}$, respectively.

The minimization of the error norm $E_2(\Omega)$ can be performed by applying any numerical optimization procedure. Afterwards, an optimized set of Q model parameters is obtained: $\Omega = (0.10391, 0.030488, 1103.4298, 1091.5206)$. Based on these values, the parameters for our deterministic simulation model can also be determined.

The complementary CDF, normalized LCR and ADF of the Q model, simulation model and measurement data are shown in Figs. 1–3, where the results corresponding to the Rayleigh model are also plotted for reasons of comparison. The analytical results are attained by using the optimized parameters given in Fig. 2, and by selecting the values of N_1 and N_2 to be 10 and 11, respectively. The simulation results obtained from the channel simulator are also presented to validate the correctness of the analytical results. From these figures, it can be seen that the analytical and simulation results of the Q model are in excellent conformity with the corresponding measurements. Due to the fact that the inphase and quadrature components describing the Q process are allowed to have different variances and different maximum Doppler frequencies, the flexibility according to the statistical properties (CDF, LCR, and ADF) of this model can considerably be increased compared with the Rayleigh model. This conclusion is proved by the fact that the Q model enables a better statistical adaptation to experimental measurement data than the Rayleigh model, as demonstrated in Figs. 1–3. The analytical and simulated PDFs of the amplitude and phase of the Q and Rayleigh models are compared in Figs. 4–6. Again, a remarkable good agreement between the first order statistics of the simulation model and those of the underlying reference model is observed, thus, demonstrating the ability of our channel simulator to approximate the Q model with very high precision.

CONCLUSION

In this paper, we have investigated the statistical properties of deterministic simulation models for Q fading channels. The analytical expressions for the PDF of the amplitude and phase, as well as for the second order statistics, the LCR and ADF, have been derived. The described deterministic simulation model for the Q model is essentially the same as that used for the Rayleigh model, except that the in-phase and quadrature components of the Q process can have different variances and different maximum Doppler frequencies. By various analytical and simulation results, the excellent conformity of the statistical properties of the Q model with the corresponding simulation model has been observed. Furthermore, the usefulness of the statistical Q model and the deterministic simulation model has been demonstrated by adapting the complementary CDF, normalized LCR and ADF to those of measurement data. Due to the increased flexibility, the Q model is shown to be in a better coincidence with measurement data than the Rayleigh model.

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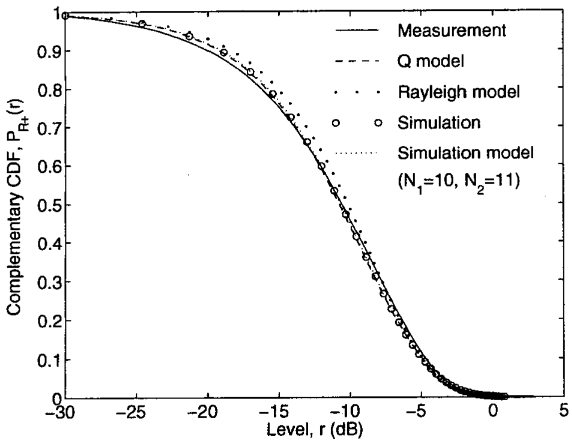

Fig. 1: The complementary CDF $P_{R+}(r)$ for the heavy shadowing environment.

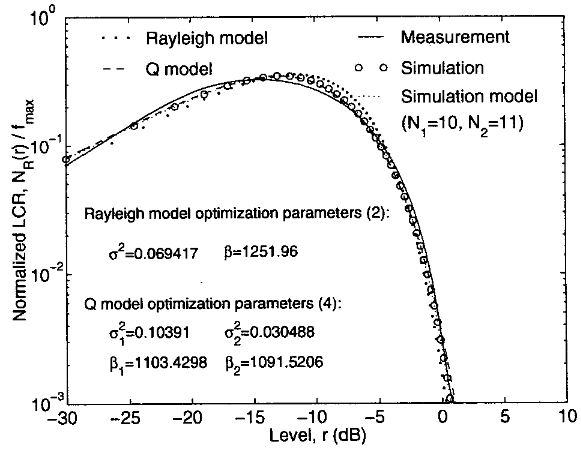

Fig. 2: The normalized LCR $N_R(r)/f_{max}$ for the heavy shadowing environment.

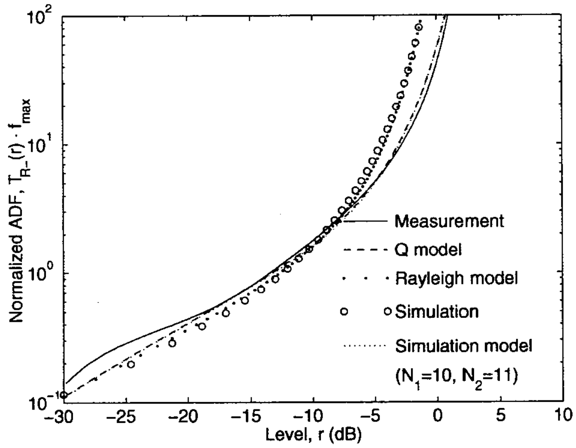

Fig. 3: The normalized ADF $T_{R-}(r) \cdot f_{max}$ for the heavy shadowing environment.

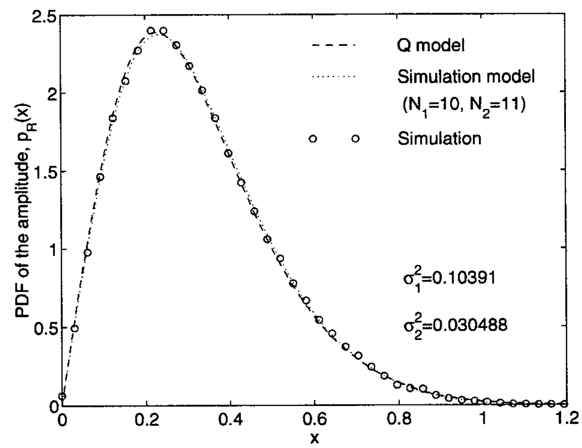

Fig. 4: Amplitude PDF $p_R(x)$ (Q model) in comparison with $\tilde{p}_R(x)$ (simulation model, $N_1 = 10$, $N_2 = 11$).

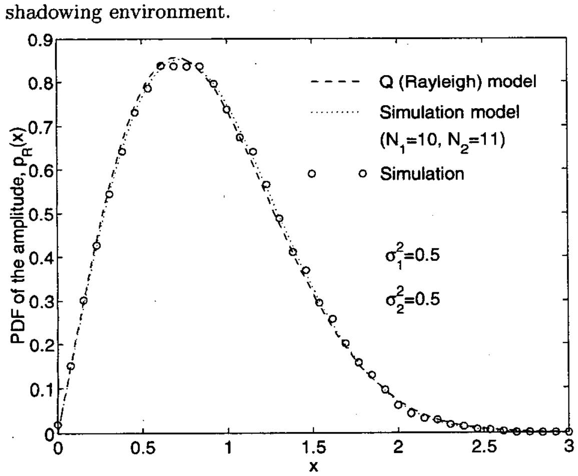

Fig. 5: Amplitude PDF $p_R(x)$ for the Q (Rayleigh) model in comparison with $\tilde{p}_R(x)$ (simulation model, $N_1 = 10$, $N_2 = 11$).

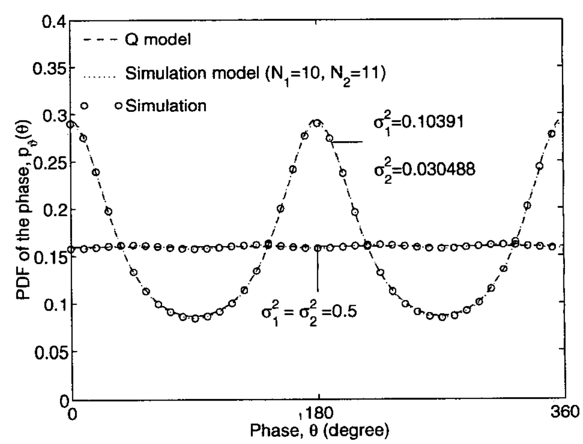

Fig. 6: Phase PDF $p_\vartheta(\theta)$ (Q model) in comparison with $\tilde{p}_\vartheta(\theta)$ (simulation model, $N_1 = 10$, $N_2 = 11$).