Heriot-Watt University
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Tutorial 3
Digital Signal Processing

1. Using the definition of a four point discrete Fourier transform, derive from first principles the radix 2 decimation in time algorithm and show its flowgraph including the multiplier weights.
2. Using the flow graph from question 1 calculate the four point DFT of the sequence x(n) = {3, 2, 1, 0}.
3. A real valued sequence x(n) has M_1 samples. The data are passed through a digital filter whose unit sample response h(n) is M_2 samples in duration. It is desired to find y(n), the linear convolution of x(n) and h(n). Consider the mathematical complexity of two approaches for doing this.

(a) The direct convolution of h(n) and x(n). Determine the number of real multiplies that would be required if M_1=600 and M_2=400.

(b) Calculating the appropriate DFTs of x(n) and h(n) using the radix 2 FFT algorithm. Then calculating y(n) with an inverse transform. Determine the number of real multiplies that would be required if M_1=600 and M_2=400.

4. (a) Derive the expression for the coefficients of a non-recursive filter which is to approximate this frequency response:

$$H\left(e^{j\theta}\right)=1 \text{ when } 0 \le \theta \le \frac{\pi}{2} \quad ; \quad H\left(e^{j\theta}\right)=0 \text{ elsewhere}$$

(b) Calculate the values of the first 13 coefficients from the expression derived in (a).

(c) Derive an expression for $\hat{H}\left(e^{j\theta}\right)$ and either attempt to sketch or plot out with the aid of a computer its spectrum.

5. Using the expression derived in the previous question, determine the modified filter coefficients for the Hamming window function. The Hamming window is defined as:

$$W_H(n) = 0.54 + 0.46\cos\left(\frac{n\pi}{N}\right) \text{ for } 0 \le n \le N$$

What function does the Hamming window perform?